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Nonlinear Vibration Phenomena in Films of Solar Arrays

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Analytical investigations of nonlinear vibration phenomena in films caused by local compression are presented. The equations of motion for a moving wrinkle are investigated and exact solutions for the longitudinal and transverse vibrations of a wrinkling film are derived.

Introduction

THE state-of-the-art for structural modeling of large flexible solar arrays such as are proposed for the Comet Rendezvous Mission has some important limitations. Tests of prototype structures have shown a nonlinear behavior of such arrays which cannot be predicted by conventional linear models because the linear plane strain (membrane) finite element model allows compression, where in reality the thin film does not allow such compression but rather forms wrinkles in the direction orthogonal to the principal direction of compression. The complicated behavior of moving wrinkles explains nonlinear phenomenon occurring in films such as shocks and snaps. The static problem of wrinkling has been considered previously.[1-4] Some dynamic effects such as the instability of the surface shape and the formation of snaps were developed.[5,6] A general theory of wrinkling of films has also been developed.[7] This article is devoted to the investigation of nonlinear effects caused by the behavior of a single wrinkle. Such a wrinkle is identified by analogy with a string having a variable mass caused by the addition or loss of film particles. Although the results presented are for an inextensible film they are valid qualitatively for extensible films.

In the context of this paper, shock is defined as a mathematical step function in a parameter such as displacement or velocity. The physical phenomenon is referred to as a jump or a snap, the latter describing the transition from a slack to a stretched state of a film or string. A reflection is a sign reversal of a parameter analogous to that used in the theory of wave propagation.

Transverse In-Plane Oscillations

Let us consider an inextensible film of a solar array which is stretched between two parallel supports AB and CD as shown in Fig. 1.

Assume that the film is attached to a rigid support AB and wound on a drum CD which provides a constant tension, T_0. Let the free edge AC deform as shown in Fig. 1. Assume this deformation u to be small such that the elongation along AC can be neglected analogous to string theory. Since in the initial state the film is subjected to tension only in the vertical direction of Fig. 1 and such deformation leads to contraction or compression of its transverse elements, it can result in the instability of the film shape. It was shown that the contraction cannot propagate in space in front of the source of the disturbance, and can only coincide with the disturbance.[7] Hence the deforming edge AC, as a singular boundary wrinkle, will absorb new particles of the contracting film and its motion can be described by the governing equation of a string with variable mass[6]:

$$\rho\frac{\partial^2 u}{\partial t^2} - T_0^*\frac{\partial^2 u}{\partial x^2} = F_R \tag{1}$$

$$T_0^* = T_0\frac{\rho}{\rho_0}$$

where ρ_0 is the initial film density, ρ the density of the moving wrinkle, T_0 the film tension, u the transverse in-plane displacement, x the coordinate of the film length, t the time, and F_R the reactive force produced by the variable mass.

From the theory of bodies with variable mass[8]:

$$F_R = \frac{d\rho}{dt}v \tag{2}$$

where v is the velocity of the particles being absorbed with respect to the moving mass. It is clear that

$$v = -\frac{\partial u}{\partial t} \tag{3}$$

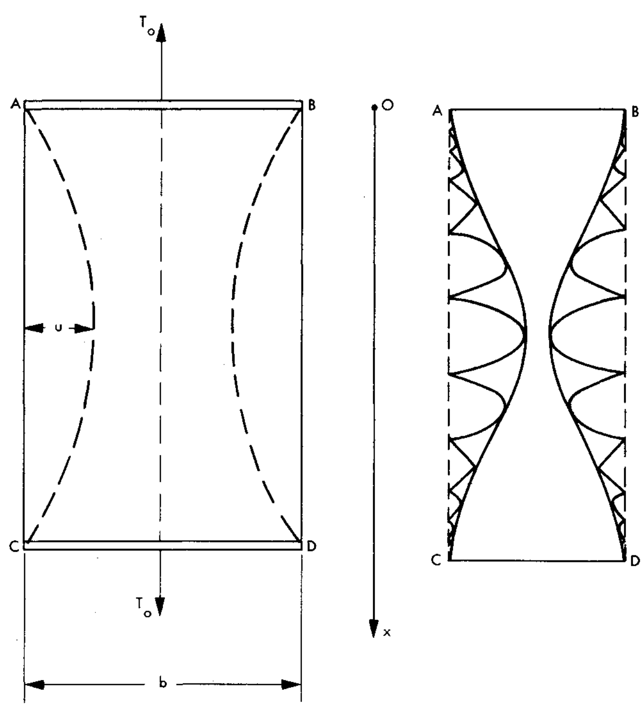

Fig. 1 Transverse in-plane deflections of stretched thin film.

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$$\rho = \rho_0 u \tag{4}$$

Consequently,

$$F_R = -\rho_0 \left(\frac{\partial u}{\partial t}\right)^2 \tag{5}$$

From the original Eqs. (1) and (5) the following can be derived:

$$u\frac{\partial^2 u}{\partial t^2} + \left(\frac{\partial u}{\partial t}\right)^2 = a^2 u \frac{\partial^2 u}{\partial x^2} \tag{6}$$

$$a^2 = \frac{T_0}{\rho_0}$$

The solution of Eq. (6) can be expressed in the following form:

$$u = \theta X, \quad \theta = \theta(t), \quad X = X(x) \tag{7}$$

Separating variables,

$$X'' + \lambda X = 0 \tag{8}$$

$$\theta\ddot{\theta} + \dot{\theta}^2 + a^2\lambda\theta^2 = 0 \tag{9}$$

where the constant λ must be determined from the boundary conditions. Equation (9) together with the boundary conditions

$$X(o) = 0, \quad X(\ell) = 0 \tag{10}$$

leads to the following eigenvalues:

$$\lambda_n = (\pi n/\ell)^2, \quad n = 1,2,\ldots \tag{11}$$

Thus,

$$X_n = \sin(\pi n/\ell)x \tag{12}$$

substituting Eq. (11) into Eq. (9):

$$\theta_n\ddot{\theta}_n + \dot{\theta}_n^2 + \alpha_n^2\theta_n^2 = 0 \tag{13}$$

$$\alpha_n = \frac{T_0}{\rho_0}\left(\frac{\pi n}{\ell}\right)^2$$

writing this equation in the form:

$$(\ddot{\theta}_n^2) + 2\alpha_n^2\theta_n^2 = 0 \tag{14}$$

the following solution can be obtained:

$$\theta_n = \sqrt{|A_n \sin\alpha_n\sqrt{2}t + B_n \cos\alpha_n\sqrt{2}t|} \tag{15}$$

where A_n and B_n are constants.

Thus any function

$$u_n = \sin(\pi n/\ell)x \cdot \sqrt{|A_n \sin\alpha_n\sqrt{2}t + B_n\cos\alpha_n\sqrt{2}t|} \tag{16}$$

is a solution of the original Eq. (1). However, because of the nonlinearity of that equation the superposition of Eq. (16) is, generally speaking, not necessarily a solution. This means that in the general case when the initial deformation cannot be represented in the form:

$$u_n(t=0) = C\sin(\pi n/\ell)x \, (n = 1,2,\ldots) \tag{17}$$

where C is a constant, the method of separation of variables is not applicable.

Let us consider Eq. (16) at $n=1$. Setting the initial conditions in terms of the impulse $I_0 \sin\pi/\ell\, x$ and the kinetic energy $W_0 \sin\pi/\ell\, x$ of the initial disturbance in the form of a shock, we can define constants A_l and B_l as,

$$A_l = \sqrt{2}I_0/\alpha_l, \quad B_l = I_0^4/4W_0^2 \tag{18}$$

Consequently,

$$u_{t=0} = I_0^2/2W_0, \quad \dot{u}_{t=0} = 2W_0/I_0 \tag{19}$$

This means that at the time of the initial shock neither the displacement nor the velocity are equal to zero. In other words, in the course of the initial shock both the displacement and the velocity change abruptly:

$$\Delta u = I_0^2/2W_0, \quad \Delta\dot{u} = 2W_0/I_0 \tag{20}$$

This effect follows from the fact that the mass of the wrinkle tends to zero if $u \to 0$ and the original Eq. (1) has a singularity. The subsequent motion can be described by the expression:

$$u = \sin\frac{\pi}{\ell}x \cdot \sqrt{\left|\frac{\sqrt{2}I_0}{\alpha_l}\sin\sqrt{2}\alpha_l t + \frac{I_0^4}{4W_0^2}\cos\sqrt{2}\alpha_l t\right|} \tag{21}$$

This solution possesses some features typical of harmonic oscillations, shown in Fig. 2. These are:

1) The amplitude of the oscillation at any fixed x is constant and defined only by the initial conditions:

$$u_{\max} = \sqrt[4]{\frac{2I_0^2}{\alpha_l^2} + \frac{I_0^8}{16W_0^4}}\sin\frac{\pi}{\ell}x \tag{22}$$

2) The period of oscillation (interval between two shocks) is also constant:

$$\tau = \arccos\frac{I_0^3\alpha_l - 4\sqrt{2}W_0^2}{I_0^3 + 4\sqrt{2}W_0^2} \tag{23}$$

But at the same time the solution has some nonlinear features:

1) The period of oscillation according to Eq. (23) depends upon the initial conditions and consequently, on the amplitude.

2) At the time

$$t^* = t + \tau K \, (K = 1,2,\ldots) \tag{24}$$

there are shocks with velocities:

$$\dot{u} = \frac{4W_0}{I_0}\sin\frac{\pi}{\ell}x \tag{25}$$

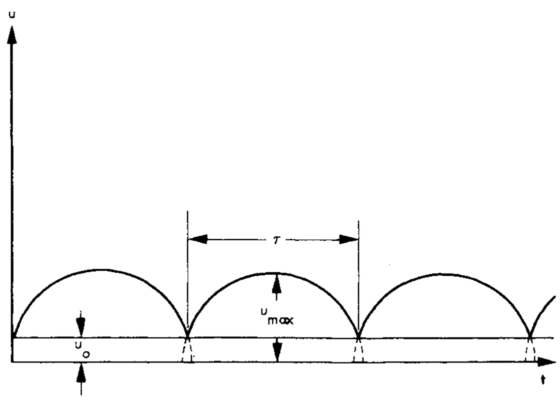

Fig. 2 Oscillations of selected particle.

as a result of a reflection from the line defined by

$$u = \frac{I_0^2}{2W_0}\sin\frac{\pi}{\ell}x \tag{26}$$

3) At all times the zone

$$u \leq \frac{I_0^2}{2W_0}\sin\frac{\pi}{\ell}x \tag{27}$$

approaches but never reaches the film width b and the line defined by Eq. (26) forms the edge wrinkle with mass

$$m = \rho_0 \frac{I_0^2}{2W_0}\sin\frac{\pi}{\ell}x \tag{28}$$

4) The harmonic frequencies ν after expansion of the solution in Fourier series will be multiples of the main frequency:

$$\nu = \pi/\tau \tag{29}$$

which depends upon the initial conditions according to Eq. (23).

Some remarks about the general case of the initial conditions can be made. First of all, note that if Eq. (16) is the particular solution of Eq. (6), then the function:

$$u = \sum_n \sin\frac{\pi n}{\ell}x \cdot \sqrt{\left| \sum_n (A_n \sin\alpha_n\sqrt{2}t + B_n\cos\alpha_n\sqrt{2}t) \right|} \tag{30}$$

$$u_{\max} > I_0/2W_0$$

will also be a solution of the above equation. Applying the initial conditions one obtains:

$$u(x,0) = \sum_n \sin\frac{\pi n}{\ell}x\sqrt{\left|\sum_n B_n\right|} \tag{31}$$

$$\dot{u}(x,0) = \sum_n \sin\frac{\pi n}{\ell}x\left(\sum_n \alpha_n B_n \Big/ \sqrt{2\left|\sum_n B_n\right|}\right) \tag{32}$$

Hence if the given initial conditions

$$u(x,0) = f(x),\; u(\dot{x},0) = \psi(x) \tag{33}$$

can be approximated in the form of Eqs. (31) and (32), then the solution of Eq. (6) can be written in the form of Eq. (30). Note that the approximations of Eqs. (31) and (32) differ from the Fourier series expansion, therefore strictly speaking, it is not an exact solution for the general case.

The solutions of Eq. (16), (21), or (30) describe the symmetric oscillations of the edges AC and BD with respect to the centerline (Fig. 1). The time of the reflections t^* corresponds to the time when the distance between these edges is maximum. The asymmetric oscillations differ from the above case only by the time of occurrence and by the location of the shocks. For instance, if the oscillations of the film edges are described by:

$$u_i = \sin(\pi n_i/\ell)x \cdot \sqrt{|A_i \sin\sqrt{2}\alpha_i t + B_i \cos\sqrt{2}\alpha_i t|} \tag{34}$$

$$u_i \geq I_0^{(i)}/2W_0^{(i)} \; (i=1,2) \tag{35}$$

the shock will appear at the time when

$$u_2 - u_1 = b - \sum_{i=1}^{2} \frac{I_0^{(i)}}{2W_0^{(i)}} \tag{36}$$

New coefficients A' and B' after the shock are easily defined from the balance of the impulse and the kinetic energy before and after the shocks.

In the particular case when

$$u_2 - u_1 \equiv b \tag{37}$$

the oscillations are without contraction. Instead of Eq. (6) one now obtains:

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} \tag{38}$$

i.e., the classical equation for transverse oscillations.

Note that in the general case when the initial conditions are different from those of Eq. (17) and every particle has its own timing for the shocks, it is possible to introduce the speed of the shock propagation along the length of the film.[7]

$$\lambda' = \sqrt{T_0/\rho_0} = a \tag{39}$$

Thus in this case we arrive at additional longitudinal shocks oscillating with the period.

$$T' = 2\ell/a \tag{40}$$

Longitudinal Oscillations in Films of Rotating Solar Sail

Consider an inextensible film of a solar sail rotating with the angular velocity ω (Fig. 3). Assume that the film is attached along line AA and the end is free. The film can be considered, with a sufficient degree of accuracy, as uniformly stretched along its length and hence can be treated as an inextensible flexible thread.

Let the free end BB be deformed uniformly with respect to the film width. This deformation will lead to a contraction of the film elements. This contraction leads to a wrinkle formation as a result of an instability of the film shape which cannot propagate along the film ahead of the source of the disturbance.[7] In the course of this deformation or crumpling the line BB will absorb new film particles and move toward

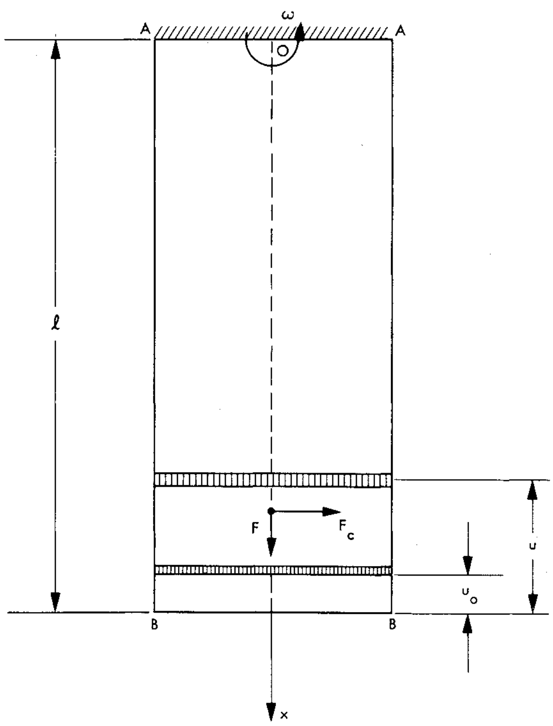

Fig. 3 Longitudinal deflections in rotating film blade.

the fixed edge AA. Invoking the theory of particles with variable mass the following governing equation is obtained

$$\frac{d}{dt}\left(m\frac{du}{dt}\right)=F, \quad u\geq 0 \tag{41}$$

where m is the variable mass of the moving "wrinkle," u the displacement of the above "wrinkle," and F the external force.

Introducing the linear density of the film

$$m=\rho u, \quad F=\rho\omega^2 u(\ell-u) \tag{42}$$

where ℓ is the length of the film. Thus instead of Eq. (1) one obtains,

$$\frac{d}{dt}\left(u\frac{du}{dt}\right)=-\omega^2 u(\ell-u) \tag{43}$$

or

$$u\ddot{u}+\dot{u}^2+\omega^2 u(\ell-u)=0$$

Introducing a new variable:

$$\dot{u}^2=Z \tag{44}$$

the following equation is obtained

$$\frac{dZ}{du}+2\frac{Z}{u}+2\omega^2 u(\ell-u)=0 \tag{45}$$

Hence

$$\dot{u}=\sqrt{(C/u^2)-\omega^2(\tfrac{2}{3}u\ell-\tfrac{1}{2}u^2)} \tag{46}$$

where C = arbitrary constant

Taking into account that

$$\dot{u}=0 \text{ if } u=u_{\max} \tag{47}$$

$$\dot{u}=\frac{\omega}{\tilde{u}}\sqrt{\tfrac{2}{3}\ell u_{\max}(1-\tilde{u}^3)-\tfrac{1}{2}u_{\max}^2(1-\tilde{u}^4)} \tag{48}$$

where

$$\tilde{u}=u/u_{\max} \tag{49}$$

Equation (48) defines the solution $u(t)$ in terms of elliptic functions:

$$t=\frac{1}{\omega}\int_0^{\tilde{u}}\frac{\tilde{u}d\tilde{u}}{\sqrt{\tfrac{2}{3}\ell u_{\max}(1-\tilde{u}^3)-\tfrac{1}{2}u_{\max}^2(1-\tilde{u}^4)}} \tag{50}$$

It follows from Eq. (48) that the solution has singularities at $u=0$:

$$\dot{u}\to\infty \text{ if } u\to 0 \tag{51}$$

which express the reflections from the free end. Following the argument used previously one arrives at a similar situation: at the time of the initial shock the displacement and velocity are defined by Eq. (19), the subsequent jumps in velocity at the time of reflections from the point

$$u=I_0^2/2W_0 \tag{52}$$

are defined by the formula:

$$\Delta\dot{u}=2W_0/I_0 \tag{53}$$

In general the solution to Eq. (50) looks like the solution to Eq. (21) for a fixed x shown in Fig. 2. The interval between shocks can be evaluated by means of Eq. (50) by considering the inequalities:

$$I_0^2 2w_0 \ll u_{\max} \ll \ell \tag{54}$$

$$\tau=\frac{\sqrt{u_{\max}}}{\omega\sqrt{\tfrac{2}{3}\ell}}\int_0^1\frac{\tilde{u}d\tilde{u}}{\sqrt{1-\tilde{u}^3}}=\frac{1}{\omega\sqrt{\tfrac{2}{3}u_{\max}}}\cdot\frac{1}{\pi}\frac{\sqrt{3}}{4\sqrt{4}}\{\Gamma(2/3)\}^2 \approx\frac{0.89}{\omega}\sqrt{\frac{u_{\max}}{\ell}} \tag{55}$$

Thus the frequencies of harmonic oscillation which result from the expansion in Fourier series are multipliers of the frequency $\nu=2\pi/\tau$ and hence depend on the initial conditions.

Transverse In-Plane Oscillations in Films of a Rotating Solar Sail

The longitudinal oscillations defined above in the film of a rotating solar sail lead to additional Coriolis forces, F_c.

$$F_c=\pm 2\rho u\dot{u}\omega \tag{56}$$

which act in plane and are orthogonal to the longitudinal axis of the film. These forces lead to transverse in-plane oscillations of the film.

Following a similar argument as above it is observed that contrary to the previous discussion wherein the film tension was constant, the film tension for the transverse in-plane oscillations of a blade is variable and is given by

$$T=(\rho_0/2)\omega^2\ell^2(1-\bar{x}^2), \quad \bar{x}=x/\ell \tag{57}$$

where x is the longitudinal coordinate with origin 0 at the center of rotation. Therefore, instead of Eq. (6) the governing equation now becomes

$$u\frac{\partial^2 u}{\partial t^2}+\left(\frac{\partial u}{\partial t}\right)^2=u\frac{\partial}{\partial x}\left(\frac{T}{\rho_0}\frac{\partial u}{\partial x}\right) \tag{58}$$

or

$$u\ddot{u}+\dot{u}^2=u[\tfrac{1}{2}\omega^2\ell^2(1-\bar{x}^2)u''-\omega^2\ell^2\bar{x}u'] \tag{59}$$

Separation of variables leads to Eqs. (14) and (15) with $a=\omega\ell$ while the eigenvalues α_n are defined by the Legendre equations:

$$(\bar{x}^2-1)X''+2\bar{x}X'-\lambda_n X=0, \quad \alpha_n=\omega^2\ell^2\lambda_n \tag{60}$$

instead of Eq. (2). Assuming that the function $X(\bar{x})$ is bounded within the interval $0<\bar{x}<1$ we arrive at the sequence of the eigenvalues:

$$\lambda_n=n(n+2), \quad (n=1,2...)$$

and normalized eigenfunctions:

$$X_n=\sqrt{n+\tfrac{1}{2}}P_n(\bar{x}) \tag{61}$$

where $P_n(\bar{x})$ are the Legendre polynomials. The solution to Eq. (59) can be written in the form similar to Eq. (30):

$$u=\sum_n[\sqrt{n+\tfrac{1}{2}}P_n(\bar{x})]\cdot\sqrt{\left|\sum_n(A_n\sin\sqrt{2}\alpha_n t+B_n\cos\sqrt{2}\alpha_n t)\right|} \tag{62}$$

$$u_{\max}\geq I_0/2W_0$$

If one of the eigenfrequencies of Eq. (62) is equal to any eigenfrequency of Eq. (50), after expanding the above solution in Fourier series, the resonance which arises between the longitudinal and the transverse in-plane oscillations is caused by the Coriolis force.

In the general case the velocity of shock propagation along the length of the film can be obtained from:

$$\lambda' = \frac{\sqrt{2}}{2}\omega\ell\sqrt{1-\bar{x}^2} \tag{63}$$

The equation of motion for the shock along the film is the solution to the following differential equation:

$$\lambda' = \pm\ell\frac{d\bar{x}_*}{dt} = \pm\frac{\sqrt{2}}{2}\omega\ell\sqrt{1-\bar{x}_*^2} \tag{64}$$

where $\bar{x}_*$ is the coordinate of the moving shock with initial value $\bar{x}^0_*$, consequently:

$$\bar{x}_* = \pm\tanh\left(\frac{\sqrt{2}}{2}\omega t\right) + \bar{x}^0_* \tag{65}$$

Thus the shock waves in this case move aperiodically with $\tau\to\infty$. In other words, any shock moving along the film is being retarded and does not experience a reflection from the end of the film.

A similar phenomenon with singularities is experienced when out-of-plane oscillations are studied analogous to previously reported results.[6]

Effect of Damping in Films of a Rotating Solar Sail

Next, the effect of damping on the longitudinal oscillations in the film of a rotating solar sail will be examined. In Fig. 3 assume that the film is set in motion by a radial shock applied at the fixed end and directed outward toward the free end. Such a shock will be distributed instantaneously and uniformly along the entire length of the film due to the assumed inextensibility of the film. This leads to a uniformly distributed velocity of reflection, v_0.

$$v_0 = I_0/\rho\ell \tag{66}$$

where I_0 is the initial impulse. The governing equation for the moving film with decreasing length can be written in the following form:

$$\rho_0 x\ddot{x} = \rho_0\omega^2(x^2/2) \quad \text{or} \quad \ddot{x} = \tfrac{1}{2}\omega^2 x \tag{67}$$

where the x coordinate of the film tip describes the translational motion of the entire film. Obviously, the solution for the first period of motion ($\dot{x}<0$) is:

$$x = \ell\cosh\frac{\omega}{2}t - \frac{2v_0}{\omega}\sinh\frac{\omega}{2}t \tag{68}$$

This solution is valid until

$$t = t^* = \frac{2}{\omega}\,\text{arc tanh}\frac{2v_0}{\omega\ell} \;(\text{when } \dot{x}=0) \tag{69}$$

At that time the length of the film will be minimum

$$x_{\min} = \ell\sqrt{1-\frac{4v_0^2}{\ell^2\omega^2}}, \quad v_0 < \frac{\omega\ell}{2} \tag{70}$$

while the length $(\ell - x_{\min})$ will be absorbed by the wrinkle in the root of the film. The governing equation for the moving film with increasing length has another form:

$$\rho_0 x\ddot{x} + \rho_0\dot{x}^2 = \rho_0(\omega^2/2)x^2 \tag{71}$$

or

$$(\ddot{x}^2) = \tfrac{1}{2}\omega^2\ell^2$$

The difference between Eqs. (67) and (71) comes from the fact that during the first period the relative velocity of the separating particles is zero, but the relative velocity of the joining particles during the second period is not zero. The solution of Eq. (71) is:

$$x = \ell\sqrt{1-\frac{4v_0^2}{\ell^2\omega^2}}\sqrt{\cosh\frac{\omega}{2}t}, \quad x\le\ell \tag{72}$$

This solution is valid until:

$$t \le t^{**} = \frac{2}{\omega}\,\text{arc cosh}\frac{1}{1-(4v_0^2/\ell^2\omega^2)} \;(\text{when } x=\ell) \tag{73}$$

At that time the velocity is determined by the expression:

$$v_0^{(1)} = v_0\sqrt{\tfrac{1}{2}\left(1-\frac{2v_0^2}{\ell^2\omega^2}\right)} < v_0 \tag{74}$$

The period of such a complete oscillation is:

$$\tau = t^* + t^{**} = \frac{2}{\omega}\left[\text{arc tanh}\frac{2v_0}{\omega\ell} + \text{arc cosh}\frac{1}{1-(4v_0^2/\ell^2\omega^2)}\right] \tag{75}$$

This reflection from the unwrinkled state of the film at the time $t^* = t^{**}$ results in the same state which existed at the initial $t=0$ but with another velocity $v_0^{(1)} < v_0$.

Continuing this process, the sequence of the reflection velocities is given by:

$$v_0 > v_0^{(1)} > v_0^{(2)} \ldots > v_0^{(n)} \tag{76}$$

which can be rewritten using the recurrence formula:

$$v_0^{(n)} = v_0^{(n-1)}\sqrt{\tfrac{1}{2}\left(1-\frac{2v_0^{(n-1)2}}{\ell^2\omega^2}\right)} \tag{77}$$

The intensity of the shocks at the time of their reflection can be calculated using

$$I^{(n)} = v_0^{(n)}/v_0 \quad I_0 < I^{(n-1)} \tag{78}$$

Thus the shock intensity is decreasing with time. The periods of subsequent oscillations are evaluated by

$$\tau^{(n)} = \frac{2}{\omega}\left[\text{arc tanh}\frac{2v_0^{(n)}}{\omega\ell} + \text{arc cosh}\frac{1}{1-(4v_0^{(n)^2}/\ell^2\omega^2)}\right] < \tau^{(n-1)} \tag{79}$$

Hence the frequency of oscillation is increasing with time.

The amplitudes of the oscillations are given by the expression:

$$A_n = \ell - x_{\min}^{(n)} = \ell\left[1-\sqrt{1-\left(\frac{2v_0^{(n)}}{\ell\omega}\right)^2}\right] \le A_{n-1} \tag{80}$$

Consequently the amplitudes are decreasing with time and the damping γ can be evaluated from

$$\gamma = [1-\sqrt{1-(2v_0^{(n)}/\ell\omega)^2}]/[1-\sqrt{1-(2v_0^{(n-1)}/\ell\omega)^2}] \quad (81)$$

Note that the reason for the apparent damping lies in the loss of kinetic energy during the absorption of film particles by wrinkles at the root without reflection. This is an example of absolute inelastic shock.

The above is a nonlinear phenomenon. The nonlinearity is expressed in the dependence of the period of oscillation and of the damping upon amplitude.

Conclusions

The purpose of this paper was to derive equations explaining fundamental phenomena in the vibration of thin films used in large extendible solar arrays and spinning solar sails or heliogyros. The derivations show that these phenomena exhibit nonlinear characteristics. Throughout these derivations, the thin film has been assumed inextensible and offering no resistance to compression. These assumptions are justified as first approximations in the study of basic phenomenon. The effect of extensibility introduces mathematical complexities which do not lend themselves to closed form solutions and are better investigated using numerical techniques and finite elements. The phenomena studied herein are intended as a first step in understanding the nonlinear behavior of large flexible structures as an aid in preliminary design studies.

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